# EXAMINERS REPORT

## MATHEMATICS - I

## HIGHER SECONDARY SCHOOL CERTIFICATE

### ANNUAL EXAMINATION 2018

**Compiled By**

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## PART I: STATISTICAL INFORMATION

The examination comprising a paper of 100 marks was administered to the students who had completed their one year of academic education at Higher Secondary School Certificate (HSSC) level. Question paper was divided into three sections, each containing different types of questions, namely: "Section A" consisting of question number one, having twenty compulsory structured part questions - Multiple Choice Questions (MCQs) of one mark each with 20 percent weighting; "Section B comprising fourteen open ended questions with limited cognitive demand - Short Response Questions (SRQs) of four marks each taken from the prescribed book with 40 percent weighting and candidates were required to answer any ten out of them of their own choice from this section; "Section C" consisting of seven open ended questions with greater cognitive demand - Extended Response Questions (ERQs) of eight marks each and out of which examinees were required to answer any five questions of their choice with proportionate weighting of 40 percent of the paper. Time duration of the paper was three hours.

A total of 33843 candidates appeared in this paper during the annual examination 2018 and out of them 12780 (37.76 percent) passed the examination with the grade percentage distribution as summarized in Table 1:

**Table 1: Grade-wise distribution of candidates**

| Grade | Students | Percentage of Grade |
|---|---|---|
| A1 | 34 | 0.1 |
| A | 948 | 2.8 |
| B | 1963 | 5.8 |
| C | 2132 | 6.3 |
| D | 4197 | 12.4 |
| E | 2944 | 8.7 |
| F | 20136 | 59.5 |
| Absent | 1489 | 4.4 |
| Total | 33843 | 100 |

## PART II: GENERAL COMMENTS

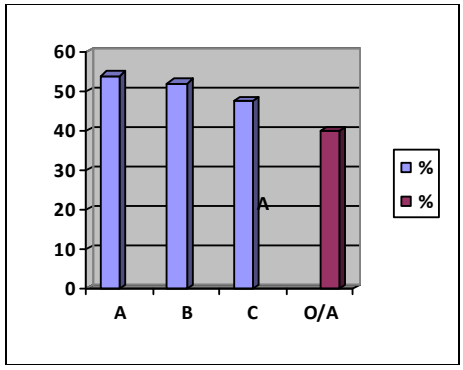
Most of the candidates handled "Section A" very well in terms of attainment of relatively the best marks when compared with sections "B and C". Generally the questions in this section comprised of mixed cognitive levels in that; 9 (45%) marks part questions were of knowledge level which required the candidates to recall different terms and procedures, 6 (30%) marks part questions required some understanding of the concepts involved and 5 (25%) marks part questions were of application level which required them to apply the concepts in the given situation. The candidates performed better in knowledge level MCQs while they faced some difficulty in answering part questions involving understanding and application of concepts. However, in 7 part questions, of paper version 1871 comprising four (2, 3, 12, 13) of knowledge, two (9, 17) of understanding and one (8) of application levels, their performance was the weakest which reflect that either majority of the candidates did not comprehend the requirements of the questions clearly or had not prepared well for the examination. The examinees secured on the average 8.31 (41.55%) marks in this section. The performance of the candidates in answering the part questions of "Section B", which consisted of three part questions 12 (21.43%) marks of knowledge, nine part questions 36 (64.29%) marks of understanding and two part questions 8 (14.29%) marks of application levels, was lower when compared with "Section A" but better than "Section C" of the paper. They generally faced difficulty in fully answering the questions which involved application of concepts, despite the fact that all questions were taken word by word from the book, which are usually discussed in the classroom by the teacher. The students secured on the average 16.45 (41.12%) marks in this section. The part questions (vi) and (xii) of understanding level from chapters on "Sequences and Series" and "Application of Trigonometry" respectively and (ix) and (xiii) of application level from chapters on "Fundamentals of Trigonometry" and "Inverse Trigonometric Functions" were the least choice questions attempted by less than 40 percent of the candidates and their performance in these questions was the lowest. This suggests that the candidates had generally carried out selective study for the examination. The "Section C" comprised of seven questions of mixed cognitive levels with a bias towards higher cognitive levels. These comprised of portions of questions of 21 (37.50%) marks of knowledge, 23 (41.07%) marks of understanding and 12 (21.43%) marks of application levels. The general performance of examinees was the lowest in this section mainly due to higher cognitive demands and they on the average secured 14.37 (35.92%) marks. The preference of candidates varied in the selection of questions and out of these seven questions, the lowest

choice three questions attempted were Question 3 by 33.79%, Question 5 by 21.55% and Question 9 by 24.79% and the three popular choice questions were Question 4 by 47.18%, Question 7 by 41.53% and Question 8 by 65.69% whereas they were required to select five out of the seven questions from this section. The overall question paper comprised of 30 (22.73%) knowledge, 61 (46.21%) understanding and 41 (31.06%) of application level portions of questions (marks). The overall performance of the candidates was satisfactory, whose performance index was 39.94 percent and had secured on the average 37.67 percent marks and almost 39.76 % passed the examination, mainly due to questions of lower level cognitive demands in the question paper, knowledge and understanding level questions. The main factor which contributed towards exaggerated reflected performance is that all questions, were given word by word from the book, which the candidates had practiced before in the classrooms and these were mere recall (knowledge level) questions for them. The section-wise performance of examinees is as indicated in Table 2 and Fig 1.

**Table 2: Accumulative performance.**

| Section | Performance Index | % |
|---|---|---|
| A | 0.54 | 54.00 |
| B | 0.52 | 52.09 |
| C | 0.48 | 47.53 |
| O/A | 0.40 | 39.94 |

**Fig. 1 Accumulative performance in all sections.**



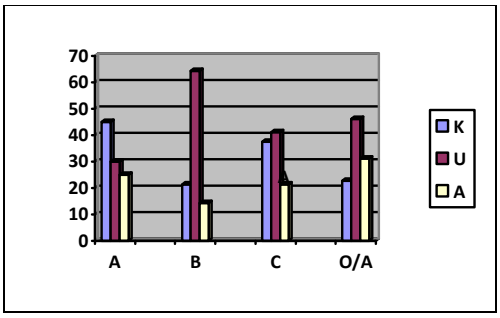

In certain questions where understanding of concepts or their application was required for answering, the candidates faced some difficulty in comprehending the requirements completely, especially in case of open ended questions contained in sections B and C. In addition, the questions from the chapters on "Sequences and Series", "Application of Trigonometry", "Fundamentals of Trigonometry" and "Inverse Trigonometric Functions" were the low choice questions attempted by the candidates irrespective of their cognitive levels which reflect that these chapters were not well prepared by them and had carried out selective study for the examination. Some of the candidates who attempted questions from these chapters their answers were ambiguous, incomplete or irrelevant. However, in case of

recall (knowledge level) questions, they answered better due to their previous knowledge and experience. The overall performance of the examinees varied from section to section, the best in case of "Section A" (MCQs) followed by "Section B" (SRQs) and comparatively the lowest in "Section C" (ERQs). The section-wise marks allocated to different question items according to their cognitive levels are given in the Table 3 and fig 2.

**Table 3: Marks % - Cognitive Level**

| Section | Knowledge (K) | Understanding (U) | Application (A) |
|---|---|---|---|
| A | 45 | 30 | 25 |
| B | 21.42 | 64.29 | 14.29 |
| C | 37.50 | 41.07 | 21.43 |
| O/A | 22.73 | 46.21 | 31.06 |

**Fig.2 Marks % allocated in all sections.**




**Areas Noted for Improvement of Various Stakeholders**

Analysis of question paper, answer scripts, performance and observation/comments of Head and Sub-examiners led to the identification of following deficiencies in the examination system:

Question Paper

i) Question paper setter used appropriate command words while writing the question items for different sections of this paper. The question items were biased towards higher cognitive levels due to which candidates faced difficulty in answering. The performance of the candidates is just satisfactory presumably due to time constraints and selective study by the candidates for the examination.

ii) Inability of paper setter to construct questions where students were required to be tested for understanding of concepts and their application in a novel situation to discriminate those who understand the concepts and can apply them in a given situations from those who can reproduce solutions through memory only. The questions given in the paper were reproduced verbatim from the book, due to which the candidates were tested for the preparation for the examination only. The candidates, where teachers had thoroughly covered

the questions given in the book, performed better when compared with those where it was partially covered or left to the students to practice at their own time.

Student Learning Objectives and Examination Grid

iii) The question paper was prepared from the contents of the text book only and the Student Learning Objectives (SLOs) of the course were ignored; even the MCQs were reproduced. To facilitate the paper setters and the teachers in the institutions, the board had issued the SLOs of the subject and a model question paper which was to be followed in its true letter and spirit including the implied examination grid. The question paper was prepared, following the pattern of the model question paper only, without ensuring the required percentage of question items of different difficulty levels (examination grid). There is a requirement to issue the Examination Grid for this paper to all stakeholders to ensure standardization and compliance by all concerned.

Marking Key

iv) Marking Key which was issued to the sub-examiners to ensure uniformity in marking of answer scripts by them was appropriately worded and prepared with minor details of award of marks. It is felt that marking of this paper was by and large quite reliable. The Marking Key is the backbone of the entire evaluation system and needed to be elaborate explaining the requirements along with award of marks for each completely or partially answered portions of a question. The preparation of Marking Key is the responsibility of the Question Paper Setter to ensure the uniformity of marking of answer scripts keeping in view the requirements of questions given by the paper setter at all marking centres but here in the instant case it was prepared by the Head Examiner of the marking team at Islamabad centre as per the requirements ascertained by him.

Markers

v) The inability of markers to evaluate accurately the answer books is evident from the non-uniformity in award of marks in that, similar answers of different questions were awarded differently, certain incorrect steps were ignored and some incomplete answers were given full credit. This suggests incompetence of markers to comprehend fully the requirements of question items and they also ignored the details given in the marking key and largely depended upon their own experience and understanding in awarding marks. In addition, the disconnect between the question paper setter and the heads of marking teams who prepared the marking keys also contributed towards the non-uniformity in award of marks at different marking centres.

vi) The purpose of examination is twofold; one is to determine the students who have successfully acquired the needed knowledge and skills required at their level and can apply certain concepts in a novel situation and the second one is to place the successful candidates in different grades according to their attainment levels. The question paper, requiring to recall solutions of the questions as given in the text book, ignoring the time constraints and a disconnect between the question paper setter and the head examiner in preparation of marking key besides unreliable marking of answer books by inexperienced markers negate the very purpose of examination and it appears mere an exercise. The examination in its present form will promote rote learning by the students only without testing their comprehension and application skills.

Recommendations

vii) There is a requirement to change existing system of paper setting and marking procedure of answer books at the board level to ensure validity and reliability of the examination. Parameters (question paper pattern and examination grid) be laid down as per international standards of each subject and must be followed for at least five years to ascertain and compare accurately the progress made by the students in teaching and learning in the institutions, against a uniform assessment standard. In the present situation, in the absence of a reliable examination system progress shown through the marks and grades attained by the students remains questionable.

viii) There is a growing trend among the students to carry out selective study for the examination and in that they leave certain important chapters on choice which weakens their base for higher studies. Although their results at HSSC level improve by selective study but it is adversely affecting their professional competence in the practical life. Further, this weaker base is transferred from generation to generation. Therefore, the choice given to the candidates in "Section B" containing (SRQs to ensure proper coverage of syllabi) where they are required to attempt any ten questions out of the given fourteen should be withdrawn, as being done at international level where all such questions are compulsory. This concession is encouraging the students to leave certain chapters on choice without losing marks.

ix) Efforts should be made at the board level to improve quality of question papers in terms of their face, contents, construct and criterion validity through capacity building of test item writers and paper setters.

x) Workshops may be arranged and refresher courses be run for the sub-examiners besides inducting more competent markers available into the system using IT technology.

xi) Efforts of FBISE towards establishment and strengthening of question bank, prepared by the professionals besides continuous additions into it may help to reduce issues of composition and construct of quality question papers. As a start point question papers set by internationally known credible boards during their previous years examinations at HSSC/equivalent levels may be reviewed by a committee of experts of each subject to lay down the guiding principles for preparing the test items.

## PART III: QUESTION SPECIFIC COMMENTS

### SECTION - A

Question Number One, an exclusive question in this section consisting of 20 compulsory Multiple Choice Questions "(MCQs)" as part or sub-questions was attempted by all (99.80%) examinees present in the examination halls and their overall achievement in this section is summarized in Table 4 below:

**Table 4: Distribution of candidates against different levels of achievement**

| **Marks** | 1 – 4 | 5 – 8 | 9 – 12 | 13 – 16 | 17 – 20 | Mean (Percentage) |
|---|---|---|---|---|---|---|
| **Percentage** | 5.68 | 27.29 | 42.97 | 16.68 | 7.38 | 8.31 (48.89) |

Mean marks obtained in this section by the examinees are 8.31 (48.89%) with an overall achievement of around 54.00 percent which is approximately 2 and 7.5 percent higher than their overall performance in the succeeding sections "B and C". Overall achievement of 54.00 percent has been spread with a bulk around 87 percent of the candidates in the middle three quintiles. Around 5.68 percent of examinees are positioned in the lowest quintile of marks (less than 20%) while 7.38 percent are in highest quintile of marks (above 80%) and around 70.26 percent are positioned in the second and third quintiles of marks i.e. between 20 and 60 percent in this section.

Decomposition analysis of part questions revealed that there were nine part questions of knowledge, five part questions of understanding and six part questions of application levels. Since all questions were compulsory, without negative marking for incorrect answers, so

everyone attempted all part questions. Question and response analysis of this section is summarized in Table 5 below:

**Table 5: Question and response analysis with option chosen against each question (Version Number 1871)**

| Question | % A | % B | % C | % D | **Comments** {cognitive level (CL), discrimination index (DI), poor distracter (PD), strong distracter (SD), facility index (FI) | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | CL | DI (0.27) | FI | PD | SD |
| 1 | 13.96 | 73.85 | 5.19 | 7.00 | U | 0.45 | 0.74 | C,D | -- |
| 2 | 21.69 | 31.59 | 31.95 | 13.22 | K | 0.28 | 0.32 | -- | A,C |
| 3 | 26.04 | 17.93 | 23.30 | 30.60 | K | 0.25 | 0.31 | -- | A,C |
| 4 | 25.90 | 46.33 | 16.06 | 9.81 | K | 0.43 | 0.46 | B | A |
| 5 | 10.26 | 21.07 | 46.59 | 21.68 | A | 0.65 | 0.47 | -- | B,D |
| 6 | 18.70 | 26.72 | 17.04 | 35.14 | K | 0.56 | 0.35 | -- | B |
| 7 | 6.12 | 14.75 | 33.96 | 43.20 | K | 0.49 | 0.43 | A | C |
| 8 | 23.09 | 24.92 | 31.31 | 17.14 | A | 0.22 | 0.25 | -- | A,C |
| 9 | 19.93 | 38.26 | 22.96 | 14.65 | U | 0.14 | 0.20 | -- | B,C |
| 10 | 11.85 | 14.79 | 53.45 | 17.22 | K | 0.54 | 0.53 | -- | -- |
| 11 | 19.82 | 42.58 | 26.10 | 9.70 | U | 0.47 | 0.42 | D | A,C |
| 12 | 46.86 | 19.71 | 20.21 | 9.08 | K | 0.02 | 0.20 | D | A,C |
| 13 | 26.81 | 25.76 | 33.19 | 10.20 | K | 0.36 | 0.33 | -- | A,B |
| 14 | 39.17 | 14.38 | 20.50 | 23.13 | A | 0.40 | 0.39 | D | C,D |
| 15 | 19.77 | 42.46 | 20.68 | 11.66 | A | 0.45 | 0.42 | -- | A,C |
| 16 | 13.70 | 7.07 | 6.81 | 70.36 | U | 0.50 | 0.70 | B,C | -- |
| 17 | 23.90 | 22.09 | 22.09 | 27.24 | U | 0.21 | 0.22 | -- | A,B,D |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 18 | 17.26 | 53.90 | 15.29 | 10.04 | K | 0.55 | 0.54 | -- | -- |
| 19 | 14.77 | 23.03 | 42.03 | 15.19 | A | 0.44 | 0.42 | -- | B |
| 20 | 15.46 | 45.63 | 18.56 | 17.46 | U | 0.54 | 0.46 | -- | -- |

*The correct answer is indicated by shading
Cognitive level: Knowledge (K), Understanding (U), Application (A)

**Facility Index (FI):** It ranged between 20 and 74 percent in all part questions. The facility indices in four (20 %) part questions is between 20 and 30 percent, five (25 %) part questions is between 31 and 40 %, nine (45 %) part questions is between 41 and 60 % and in only two (10 %) part questions 16 and 1, it is 70 and 74 percent respectively.

**Discrimination Index (DI):** In order to correlate the performance of the candidates in a part question with their overall score (combined 20 part questions) discrimination index is calculated by taking 27 percent examinees each from upper and bottom sub-groups of this cohort, keeping in view the sample constraints, four different versions of question papers, in that version 1871 has been taken for analysis as it was attempted by maximum number of students 12623 (37.30 %) of the candidates. The positive value of DI, for all questions, indicates that the requirements of the questions were well understood by the examinees. On the basis of DI values, **thirteen** items are found very good test items for having values equal or more than 0.4, **one** item is reasonably good with DI value ranged from 0.30 to 0.39, **four** are marginal items with DI value ranged from 0.2 to 0.29 and **two** items are the weakest (Part questions 9 and 12 with facility index of 20 % each) with DI value 0.14 and 0.02 respectively, (Ebel and Frisbie, 1986). Amongst these, **fifteen** MCQs are found ideal questions having difficulty (facility) index range between 0.3 and 0.70 with DI value greater than 0.24.

Discrimination coefficient measuring effectiveness of each distracter has also been determined using similar formula as suggested by Nitko and Hsu (1984). Accordingly, seven part questions (2, 3, 12 and 13, of knowledge, 9 and 17 of understanding and 8 of application levels) 35 percent of MCQs were found having 2-3 effective distracters due to which both their FI is less than 0.32 and DI less than 0.24 whereas in case of four (8, 9, 12 and 17) out of them both their FI and DI are less than o.25 despite that all of them were expressed with appropriate command words stating very clearly the requirements of the questions and the

distracters which suggests that either the candidates had carried out selective study or they were not well prepared for the examination.

## SECTION – B

Question number two was the single question in this section consisting of 14 "Short Response Questions (SRQs)" as sub-questions with equal (4) marks each and the candidates were required to answer any 10 out of them. Most of the candidates, around 90-95 percent, attempted questions (iii), (v) and (xi) from the chapters on "Matrices and Determinants", "Partial Fractions" and "Trigonometric Functions and their Graphs" while among the five low choice questions, (iv), (viii), (xii), (xiii) and (xiv), were attempted by less than 50 percent of the candidates especially the last three questions from the chapters on "Application of Trigonometry", "Inverse Trigonometric Functions" and "Solutions of Trigonometric Equations". The facility index (FI) of question 2 is 0.55 and its discrimination index (DI) is 0.97. The overall performance of the candidates in this section was good where they on the average secured 16.11 (76.72 %) marks and 1.78 % out of them secured 100 % marks

Overall achievement of examinees in this section is summarized in Table 6 below:

**Table 6 : Distribution of candidates against different levels of achievement**

| **Marks** | 1 – 8 | 9 – 16 | 17 – 24 | 25 – 32 | 33 – 40 | **Mean (Percentage)** |
|---|---|---|---|---|---|---|
| **Percentage** | 3.23 | 5.66 | 14.75 | 20.74 | 55.62 | 16.11 (76.72) |

Decomposition analysis of sub-questions revealed that 21.42 percent of the questions were of knowledge, 64.29 percent of understanding and 14.29 percent of application levels. Those candidates who were good in conceptual understanding of the subject performed relatively better. Since candidates had to choose any ten sub-questions out of fourteen so there was a general trend to select questions requiring lower cognitive demand. Question and response analysis of this section is summarized (generally the most popular were "K", followed by "U" and comparatively the lowest attempted "A") in Table 7 below:

**Table 7 : Question and response analysis against each sub-question**

| Part question | (i) | (ii) | (iii) | (iv) | (v) | (vi) | (vii) | (viii) | (ix) | (x) | (xi) | (xii) | (xiii) | (xiv) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Cognitive Level (CL) | U | K | U | A | U | U | U | A | K | U | K | U | U | U |
| Attempted (%) | 75-80 | 80-85 | 90-95 | 45-50 | 90-95 | 80-85 | 70-75 | 45-50 | 50-55 | 85-90 | 90-95 | 30-35 | 45-50 | 45-50 |

**Specific Responses to parts of the Question 2:**

**Question Part (i)**

If $Z_1 = 2 + i$ , $Z_2 = 3 + 2i$, $Z_3 = 1 + 3i$, then find the value of $\frac{\overline{Z_1}.\overline{Z_2}}{Z_3}$ in form of a + bi

Question required the candidates to substituted the values of, $\overline{Z_1}$, $\overline{Z_2}$ and $Z_3$ in the equation and multiply the complex numbers in the numerator,

$$\frac{\overline{Z_1}.\overline{Z_2}}{\overline{Z_3}} = \frac{(2-i)(3-2i)}{1+3i} \quad \Rightarrow \quad \frac{\overline{Z_1}.\overline{Z_2}}{Z_3} = \frac{4-7i}{1+3i}$$

Candidates were required to multiplied both numerator and denominator by the conjugate of the denominator to have the real number in the denominator,

$$\frac{\overline{Z_1}.\overline{Z_2}}{Z_3} = \frac{4-7i}{1+3i} \times \frac{1-3i}{1-3i}$$

They were asked to express the result in the form a + bi by multiplying the complex numbers in the numerator and denominator of the equation to get the result as,

$$\frac{\overline{Z_1}.\overline{Z_2}}{Z_3} = -\frac{17}{10} - \frac{19}{10}\,i$$

*Question asked was of **understanding level** requiring the candidates to convert the complex number in the denominator into real number by multiplying both the numerator and denominator by the conjugate of the complex number in the denominator. Some candidates applied alternate methods available for the solution of this question. Almost 75-80 percent of the candidates attempted this question and majority of the answered it successfully to get the maximum marks. However, some 10–15 percent of them got confused and faced difficulty in getting the real number in the denominator. On the average they performed extremely well in this question and secured around 3.5-4 marks.*

**Question Part (ii)**

By using truth table prove that

$p \vee (\sim p \wedge \sim q) \vee (p \wedge q) = p \vee (\sim p \wedge \sim q)$

Question asked required the candidates to construct the truth table and to prove that,
L.H.S = R.H.S

| P | Q | ~p | ~q | ~p ∧ ~q | p ∧ q | R.H.S | L.H.S |
|---|---|---|---|---|---|---|---|
| T | T | F | F | F | T | T | T |
| T | F | F | T | F | F | T | T |
| F | T | T | F | F | F | F | F |
| F | F | T | T | T | F | T | T |

*Question asked was of knowledge level requiring the candidates to construct the truth table to prove that L.H.S = R.H.S. This was one of the most popular choice questions attempted by around 80-85 percent of the candidates and majority of them successfully proved the required relation except for some 15-20 percent of them who got confused. On the average students secured good marks ranging 2.5–3 marks in this question.*

**Question Part (iii)**

Show that $\begin{vmatrix} x & x & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix} = -(x+3)(x-1)^3$

This question asked required the candidates to use rules and procedures as under;
Added $(C_2 + C_3 + C_4)$ into $C_1$ on the L.H.S

$$L.H.S = \begin{vmatrix} x+3 & 1 & 1 & 1 \\ x+3 & x & 1 & 1 \\ x+3 & 1 & x & 1 \\ x+3 & 1 & 1 & x \end{vmatrix}$$

Took (x+3) common from $C_1$

$$L.H.S = (x+3)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & x & 1 & 1 \\ 1 & 1 & x & 1 \\ 1 & 1 & 1 & x \end{vmatrix}$$

Subtracted $C_1$ from other columns as;
$(C_2 - C_1)$; $(C_3 - C_1)$; $(C_4 - C_1)$

$$L.H.S = (x+3)\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & x-1 & 0 & 0 \\ 1 & 0 & x-1 & 0 \\ 1 & 0 & 0 & x-1 \end{vmatrix}$$

Opened the determinate and proved

$$L.H.S = (x+3).1.(x-1).(x-1).(x-1) = (x+3)(x-1)^3$$

*Question asked was of **knowledge-cum-understanding level** requiring the candidates to use rules and procedures to prove the required relation. This question was one of the most popular choice questions attempted by almost 90 - 95 percent of the candidates and majority of them successfully answered this question except for a few (10–15%) candidates who faced difficulty in correctly applying the rules for determinants. Some candidates adopted alternate procedures available for the solution of this question. The candidates performed extremely well in this question who on the average secured 3.5-4 marks.*

**Question Part (iv)**

Solve the equation $4.2^{2x+1} - 9.\ 2^x + 1 = 0$

This question required the candidates to simplify and reduce the equation into a quadratic equation as under,

$4.2^{2x+1} - 9.\ 2^x + 1 = 0 \quad \Rightarrow \quad 8.2^{2x} - 9.\ 2^x + 1 = 0$

Let $y = 2^x \quad \Rightarrow \quad y^2 = 2^{2x}$

$8\,y^2 - 9y + 1 = 0$

The candidates were also required to find the solution of this quadratic equation by any of the methods and the most commonly used is by factorization as under,

$\Rightarrow (y-1)(8y-1) = 0$

$\Rightarrow \ y = 1, \quad y = \frac{1}{8}$

Candidates were asked to calculate the solution set for x as under,

$y = 1 \quad \Rightarrow 2^x = 2^0 \quad \Rightarrow x = 0$

$y = \frac{1}{8} \quad \Rightarrow 2^x = 2^{-3} \quad \Rightarrow x = -3$

*Question asked was of **understanding-cum-application level** in which the candidates were required to simplify and rearrange the given equation in the form of a quadratic equation to find the solution set of $y = 2^x$ and hence the solution set of "x". Almost 45-50 percent of the candidates attempted this questions and majority (80-85%) of them successfully answered it correctly. However, some (25-25%) of the candidates faced difficulty in finding the solution set of "x" from the solution set of $y = 2^x$. On the average the candidates secured 3–3.5 marks in this question.*

**Question Part (v)**

Resolve into partial fraction $\dfrac{2x+1}{(x+3)(x-1)(x+2)^2}$

This question required the candidates to express the given equation into partial fractions as under,

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{(x+2)} + \frac{D}{(x+2)^2}$$

They were expected to multiply with the denominator of the expression on both sides of the equation as under,

$2x + 1 = A\,(x-1)(x+2)^2 + B\,(x+3)(x+2)^2 +$

$C\,(x+3)\,(x-1)\,(x+2) + D\,(x+3)\,(x-1)$

Candidates were required to calculate the values of A, B and D by putting values of,
x = -3, 1, -2

$put\ x = -3\ \ giving\ \ A = \frac{5}{4}$

$put\ x = 1\ \ giving\ \ B = \frac{1}{12}$

$put\ x = -2\ \ giving\ \ D = 1$

They were also required to calculate the value of C by equating the coefficients of $x^3$ on both sides of equation as under,

$$A + B + C = 0 \qquad \Rightarrow \quad C = -\frac{4}{3}$$

Candidates were asked to express the resolved partial fractions of the expression as under,

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{5}{4(x+3)} + \frac{1}{12(x-1)} - \frac{4}{3(x+2)} + \frac{1}{(x+2)^2}$$

*Question asked was of **knowledge-cum-understanding** level which required the candidates to recall and use the procedure to resolve the given expression into its partial fractions. This was one of the most popular choice questions attempted by almost 90-95 percent of the candidates and majority of them answered it successfully securing the maximum marks. They on the average secured 3.5–4 marks.*

**Question Part (vi)**

Find the sum to $n^{th}$ term of series

$r + (1 + k)\, r^2 + (1 + k + k^2)\, r^3 + \cdots\cdots$ n

The question asked required the candidates to suppose that,

$S_n = r + (1 + k)\, r^2 + (1 + k + k^2)\, r^3 + \cdots$ to n terms

They were expected to multiply both sides of the equation by $(1 - k)$ to convert the series into G.P. series as under,

$$(1-k)S_n = (1-k)r + (1-k)(1+k)\, r^2 + (1-k)(1+k+k^2)\, r^3 + \cdots \text{ to n terms}$$

$$(1-k)S_n = (1-k)r + (1-k^2)r^2 + (1-k^3)r^3 + \cdots \text{ to n terms}$$

$$(1-k)S_n = (r - kr) + (r^2 - k^2r^2) + (r^3 - k^3r^3) + \cdots \text{ to n terms}$$

$$(1-k)S_n = (r + r^2 + r^3 + \cdots \text{ to n terms}) - (kr + k^2r^2 + k^3r^3 + \cdots \text{ to n terms})$$

Candidates were required to apply the formula for addition of G.P. series

$$(1-k)S_n = \left[\frac{r(r^n-1)}{(r-1)} - \frac{kr(k^nr^n-1)}{(kr-1)}\right]$$

The candidates were also expected to express the sum up nth term of the given series by simplifying the equation as asked in the question as under,

$$S_n = \frac{r}{1-k}\left[\frac{(r^n-1)}{r-1} - \frac{k(k^nr^n-1)}{kr-1}\right]$$

They had the option to apply any of the available alternate methods to find the sum of the series.

*Question asked was of **knowledge-cum-understanding level** aimed to test the ability of candidates to convert the given series into G.P. series by multiplying it with (1-k). They were required to recall the formula for addition of G.P. series to get the addition of the given series up to the nth term as asked by the question. This question was one of the least choice questions attempted by around 80-85 percent of the candidates who secured on the average 3–3.5 marks.*

**Question Part (vii)**

Find the number greater than 23000 that can be formed from the digits 1,2, 3, 5, 6 without repeating any digit

Question asked required the candidates to first calculate the total numbers which can be formed by using all the five digits without repeating from the given digits as under,

*Given digits* : 1, 2, 3, 5, 6

Total numbers (using all digits) = ${}^5P_5 = 120$

Stated that in view of condition that numbers $>$ 23000 are to be counted

$\Rightarrow$ *when digit* 1 *is fixed on extreme left Numbers less than* 23000 *are* $= {}^4P_4 = 24$

Argued that numbers are $< 23000$ when digits 21 are on the extreme left

$\Rightarrow$ *when digits* 21 are *fixed on extreme left*

*Numbers less than* 23000 *are* $= {}^3P_3 = 6$

Calculated total numbers > 23000 = ${}^6P_6 - ({}^4P_4 + {}^3P_3)$

$$\textit{Numbers greater than } 23000 = 120 - (24 + 6) = 90$$

*Question asked was of **knowledge-cum-understanding level** aimed to test ability of the candidates to find the total numbers which can be formed using the given six digits without repeating and then subtracting from it the numbers not meeting the given condition. This question was attempted by around 70-75 percent of the candidates and majority of them found the total numbers which can be formed. However, some (20-25) percent out of them faced difficulty in finding the numbers not meeting the given condition. The overall performance of the candidates was good who on the average secured 2.5-3 marks.*

**Question Part (viii)**

If x is so small that its square and higher powers may be neglected then show that

$$\frac{(1+x)^{\frac{1}{2}}(4-3x)^{\frac{3}{2}}}{(8+5x)^{\frac{1}{3}}} = 4\left(1 - \frac{5x}{6}\right)$$

Question asked required the candidates to prove that when x is so small that its squares and higher powers can be neglected in the given relation so,

$$\frac{(1+x)^{\frac{1}{2}}(4-3x)^{\frac{3}{2}}}{(8+5x)^{\frac{1}{3}}} \approx 4\left(1 - \frac{5x}{6}\right)$$

They were required to simplify the L.H.S of the equation by applying the Binomial Theorem retaining the terms in x only as under,

$$\Rightarrow \quad \frac{(1+x)^{\frac{1}{2}}(4-3x)^{\frac{3}{2}}}{(8+5x)^{\frac{1}{3}}} = (1+x)^{\frac{1}{2}}(4-3x)^{\frac{3}{2}}(8+5x)^{-\frac{1}{3}}$$

$$\text{L.H.S} = 4\,(1+x)^{\frac{1}{2}}\left(1-\frac{3}{4}x\right)^{\frac{3}{2}}\left(1+\frac{5}{8}x\right)^{-\frac{1}{3}}$$

$$\text{L.H.S} \approx 4\left(1+\frac{1}{2}x\right)\left(1-\frac{9}{8}x\right)\left(1-\frac{5}{24}x\right)$$

They were required to multiply the resultant terms, retaining terms having x only to get the required relation as under,

$$\text{L.H.S} \approx 4\left(1-\frac{5}{8}x\right)\left(1-\frac{5}{24}x\right)$$

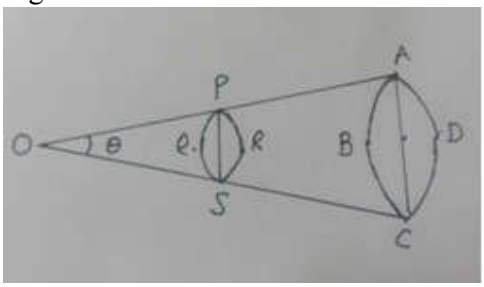
$$\text{L.H.S} \approx 4\left(1-\frac{5}{6}x\right) = \text{R.H.S}$$

*Question asked was of* ***understanding-cum-application level*** *in which candidates were required to apply the Binomial Theorem to prove the required relation when x is so small that its squares and higher powers can be neglected. This question was one of the low choice questions attempted by almost 45–50 percent of the candidates who on average secured 2-2.5 marks.*

**Question Part (ix)**

Find correct to nearest centimeter distance at which a coin of diameter 1 cm should be held so as to conceal the full moon whose diameter subtends an angle of $31'$ at the eye of observer on earth.

Question required the candidates to draw the diagram of the moon as ABCD and that of the coin PQRS as observed by eye of the observer where diameters of both moon and coin make the same angles. They took O as the observer's eye so APO and CSO are straight line segments.




Candidates were required to argue that from the diagram since both the angles subtended by the moon and coin are the same so these are equal to the angle subtended by the moon on the eye which is given as $31'$ so,

$$\text{m}\angle AOC = \text{m}\angle AOC = 31'$$

They were required to suppose that the distance of the coin from the eye $\overline{OP} = r$ and diameter of coin so, $l = 1\text{cm}$, $\theta = 31' = \frac{31\pi}{10800}$ rad

$$\because \theta = \frac{l}{r} \Rightarrow r = \frac{l}{\theta}$$

Candidates were also required to substitute the value of the diameter of the coin as given to calculate the required distance of the coin from the eye, $\overline{PS} = l = 1$ cm and

$\theta = 31' = \frac{31\pi}{10800}$ rad

$r = \frac{1 \times 10800}{31\pi} = 110.89$ cm

Distance from the eye = 110.89 cm

*Question asked was of mixed cognitive levels (knowledge, understanding and application) aimed to test ability of the candidates to recall the definition of angle in radian and to apply the concept in finding the distance between the eye and the coin when both moon and the coin overlap each other. Almost 50-55 percent of the candidates attempted this question and some (20-25%) of them tried to solve this question by adopting the alternate methods but majority (80-85%) of them were irrelevant, ambiguous and left the question incomplete. They, on the average secured 2.5-3 marks in this question.*

**Question Part (x)**

Prove that $\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} = \frac{\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}}$

This question required the candidates to apply trigonometric identities to transform the trigonometric functions on the L.H.S of the given equation in terms of half angles to complete squares as under,

L.H.S= $\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} \Rightarrow \sqrt{\frac{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}-2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}}$

They were required to simplify the terms on the L.H.S to prove the relation as asked in the question as under,

$\Rightarrow \sqrt{\frac{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)^2}{\left(\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}\right)^2}} \Rightarrow \frac{\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}} =$ R.H.S

*Question asked was of **knowledge-cum-understanding level** aimed to test the ability of the candidates to recall and use the trigonometric identities to transform the trigonometric ratios in terms of their half angles and by simplifying the equation prove the given relation. This question was one of the most popular questions attempted by almost 85–90 percent of the candidates who answered it very well except for some (20-25%) who faced difficulties in using the trigonometric identities properly to prove the relation. However, some used one of the alternate methods but majority of them were unsuccessful in proving the given relation. The candidates who attempted this question on the average secured 2.5-3 marks.*

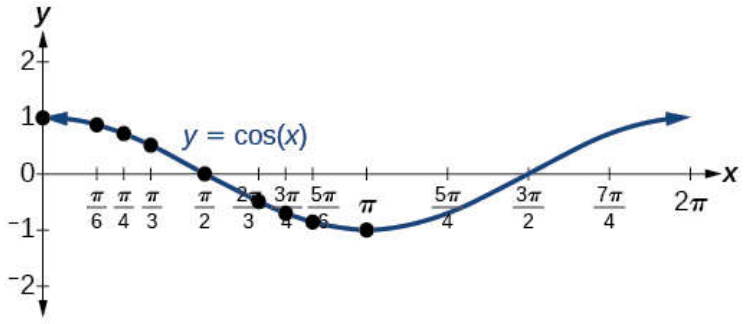
**Question Part (xi)**

Draw the graph $y = \cos x$ from 0 to $2\pi$

The question asked required the candidates to construct the table for $\cos x$ for different values of x and then plot the graph between y = cos x and x as under,

| X | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | π2 | $\frac{2\pi}{3}$ | $\frac{5\pi}{6}$ | π |
|---|---|---|---|---|---|---|---|
| y= cos x | 1 | 0.87 | 0.5 | 0 | - 0.5 | - 0.87 | - 1 |

| X | $\frac{7\pi}{6}$ | $\frac{4\pi}{3}$ | 3π2 | $\frac{2\pi}{3}$ | $\frac{5\pi}{3}$ | 2π |
|---|---|---|---|---|---|---|
| y=cosx | - 0.87 | - 0.5 | 0 | 0.5 | 0.87 | 1 |



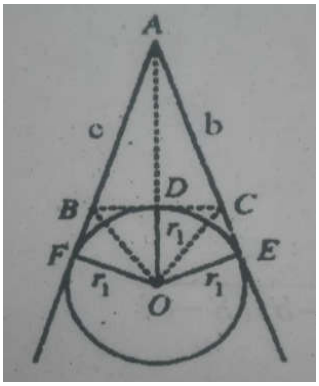

*Question asked was of **knowledge level** which required the candidates to construct yhe table for y = cos x for different values of x and then to draw graph by plotting the points as given in the table for different values of x as given in the table. This was the most popular choice question attempted by almost 90-95 percent of the candidates and majority of them drew the graph correctly. Their performance was extremely well and on the average they secured 3.5–4 marks.*

**Question Part (xii)**

By using usual notation prove that $r_1 = \frac{\Delta}{S-a}$

This question required the candidates to construct an escribed circle, having centre O opposite to the vertex A of triangle ABC with;

$\overline{AB}$ = c, $\overline{BC}$= a and $\overline{CA}$= b




They were required to draw and join O to A,B,C;

$\overline{OD} \perp \overline{BC}$

$\overline{OE} \perp \overline{AC}$ produced

$\overline{OF} \perp \overline{AB}$ produced.

They were also required to argue that since the radius of the circle remains constant so;

$\overline{OD} = \overline{OE} = \overline{OF} = r_1$

Candidates were expected to state that the area of,

$\Delta$ ABC = Area of $\Delta$ OAB + Area of $\Delta$ OAC – Area of $\Delta$ OBC

$\Rightarrow \quad \Delta = \frac{1}{2}\, c\, r_1 + \frac{1}{2}\, b\, r_1 - \frac{1}{2}\, a\, r_1$

They were required to simplify the terms to get the required equation as under,

$\Delta = \frac{1}{2} r_1 (c + b \quad a) = \frac{1}{2} r_1 (2s \quad 2a)$

$\Rightarrow \Delta = r_1(s - a) \Rightarrow r_1 = \frac{\Delta}{s-a}$

*Question asked was of **understanding level** in which candidates were required to construct an escribed circle, having centre O opposite to the vertex A of triangle ABC. Candidates were also required to prove the given relation by equating the areas of triangles formed to prove the relation as given. This was one of the low choice questions attempted by around 30–35 percent of the candidates and majority (70-75%) of them successfully proved the required relation except a few (10-15%) who were ambiguous, clueless and irrelevant in answering this question. Overall their performance was quite good who on the average secured 3–3.5 marks.*

**Question Part (xiii)**

Show that $\cos^{-1}(-x) = \pi - \cos^{-1} x$

This question required the candidates to suppose $\alpha = \cos^{-1} x$ iff $x = \cos\alpha$, and used the trigonometric identity as under,

$\because \cos(\pi - \alpha) = -\cos\alpha$

$\Rightarrow \quad \cos(\pi - \alpha) = -x$

They were required to find the angle as under,

$\Rightarrow \pi - \alpha = \cos^{-1}(-x)$

$\Rightarrow \pi - \cos^{-1}(x) = \cos^{-1}(-x)$

$\Rightarrow \quad \cos^{-1}(-x) = \pi - \cos^{-1}(x)$

Some candidates also applied the alternate methods to prove the given equation.

*Question asked was of **knowledge-cum-understanding level** in which candidates were required to recall and use the trigonometric identities and to prove the equation as asked. This was one of the low choice questions attempted by around 45–50 percent of the candidates and almost 30-35 percent out of them faced difficulties in finding the angle. The overall performance of the candidates was good who on the average secured 2–2.5 marks.*

**Question Part (xiv)**

Find the solution set of $\sin 3x + \sin 2x + \sin x = 0$

This question required the candidates to rearrange the equation and apply trigonometric identities as under,

$\sin 2x + (\sin 3x + \sin x) = 0$

$\sin 3x + \sin x = 2 \sin 2x \cos x$

$\Rightarrow \sin 2x + (\sin 3x + \sin x) = 0$

$\Rightarrow \sin 2x + 2 \sin 2x \cos x = 0$

They were expected to take $\sin 2x$ common from both terms

$\Rightarrow \sin 2x (1 + 2 \cos x) = 0$

Candidates were required to find the solution sets of the equation by taking,
Either, sin 2x = 0, or $(1 + 2 \cos x) = 0$

Calculated the values of x when, sin 2x = 0

$\Rightarrow 2x = 0, \pi \Rightarrow 2x = 0, \pi \Rightarrow 2x = 0+n\pi, \pi+n\pi \Rightarrow \Rightarrow x = n\pi, \frac{\pi}{2} + n\pi$

They were required to calculate the values of x when $1 + 2 \cos x = 0$

$\Rightarrow \cos x = -\frac{1}{2} < 0 \Rightarrow Reference\ angle = \frac{\pi}{3}$ which lies either in Quad II,

Where, $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ or in Quad III, where, $x = \pi - \frac{\pi}{3} = \frac{4\pi}{3}$

$\Rightarrow x = \frac{2\pi}{3} + 2n\pi, \frac{4\pi}{3} + 2n\pi$

Candidates were required to give the general solution as,

G.S = $\{n\pi\} \cup \left\{\frac{\pi}{2} + n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\}$

*Question asked was of **knowledge-cum-understanding level** in which candidates were required to recall the trigonometric identities and procedure for finding the solution set of the two equations constructed as a result of factorization. Candidates were expected to calculate the solution sets of each equation separately and then combine them together to get the general solution as required. This was one of the low choice questions attempted by around 45–50 percent of the candidates who on the average secured 2–2.5 marks.*

## SECTION – C

This section was comprised of seven "Extended Response Questions (ERQs)" with equal (8) marks each and the candidates were required to attempt any five out of the given seven questions. Each question consisted of mixed cognitive levels, in that Overall questions portions of 21 (37.50 %) marks were of knowledge, 23 (41.07 %) marks of understanding and 12 (21.43 %) marks of application levels. The overall achievement of the candidates in this section has been the lowest and they on the average secured 14.37 (35.92 %) marks, which is mainly due to bias of questions towards higher cognitive levels (understanding and application levels). Though all the questions were taken verbatim from the text book, which are usually covered in classroom teaching, yet the performance of students varied from question to question according to their cognitive levels. Approximately, a total of 21.25 percent examinees succeeded in attempting the five required questions from this section. Out of them around 8.43 percent attempted the highest five choice questions excluding the two lowest choice questions 3&5 from chapters on "Matrices and Determinants" and "Sequences and Series" and around 0.35 percent attempted the lowest five choice questions excluding the

two highest choice questions 4&8 from chapters on "Quadratic Equations" and "Application of Trigonometry" whereas approximately around 1.15 percent of the candidates secured 100 percent marks in this section. Overall achievement of examinees in this section is summarized in Table 4 below:

**Table 8: Distribution of candidates against different levels of achievement**

| **Marks** | 1 – 8 | 9 – 16 | 17 – 24 | 25 – 32 | 32 – 40 | **Mean (Percentage)** |
|---|---|---|---|---|---|---|
| **Percentage** | 32.22 | 26.91 | 20.32 | 12.12 | 8.43 | 14.37 (35.92) |

Decomposition analysis of the portions of the questions revealed that out of a total of 56 marks questions there were 21 (37.50%) marks portions of knowledge, 23 (41.07%) marks of understanding and 12 (21.43%) marks of application levels. Those students who were good in conceptual understanding of the subject performed relatively better. Since the candidates had to choose any five out of the given seven questions so there was a general trend to select questions requiring lower cognitive demand. Question and response analysis of this section is summarized in the table below:

**Table 9:** Question-wise Marks & % - Section C

| Question | Cognition Level | Average Marks (%) | Facility Index (FI) | Discrimination Index (DI) | Attempted (%) |
|---|---|---|---|---|---|
| 3 | K = 25 %<br>U = 25 %<br>A = 50 % | 4.02 (50.31 %) | 0.69 | 0.42 | 33.79 |
| 4 | K = 25 %<br>U = 75 % | 3.64 (45.51 %) | 0.64 | 0.44 | 47.18 |
| 5 | K = 37.5 %<br>U = 25 %<br>A = 37.50 % | 5.56 (69.48 %) | 0.79 | 0.47 | 21.55 |
| 6 | K = 25 %<br>U = 75 % | 6.28 (78.48 %) | 0.87 | 0.49 | 38.28 |

| | | | | | |
|---|---|---|---|---|---|
| 7 | K = 50 %<br>U = 25 %<br>A = 25% | 4.86 (60.71 %) | 0.70 | 0.53 | 41.53 |
| 8 | K = 25 %5<br>U = 37.5 %<br>A = 37.5 % | 4.85 (60.67 %) | 0.74 | 0.63 | 65.69 |
| 9 | K = 75 %<br>U = 25 % | 4.57 (57.07 %) | 0.74 | 0.57 | 24.79 |

**Facility Index (FI):** It ranged from 0.64 to 0.87 in all questions. Facility index of question number 3, involving the solution of system of equations using matrices was the lowest 0.64. FI above 0.60 suggests that the candidates performed extremely well in all the questions of this section.

**Discrimination Index (DI):** In order to correlate the performance of the candidates in questions (3-9) of "Section C" with their overall score (in questions 2-9) discrimination index is calculated by taking 27 percent examinees each from upper and bottom sub-groups of this cohort, keeping in view the sample constraints, results of question 1 ("Section A") and questions (2-9) received from two different sources and overall higher FI. The positive value of DI, for all questions, indicates that the requirements of the questions were well understood by the examinees. On the basis of DI values all questions are found very good test items for having values equal or more than 0.4. The DI value of combined "Section C" is 0.97, due to three reasons; first one is that the score of Question 1, was excluded from the overall score as it was received from a different source other than that from whom score of other questions was received, the second one is that the overall almost equal facility and discrimination indices of "Sections B & C" which are (0.55 & 0.47) and their discrimination indices (0.97 & 0.97) and third one was that the combined contribution of the scores of "Sections B&C" were 50 percent each of the overall score used for calculations.

**Specific Responses to Questions – Section C.**

**Question 3**

Use the matrices to solve the system of equations

$$2x_1 + x_2 + 3x_3 = 3$$
$$x_1 + x_2 - 2x_3 = 0$$
$$-3x_1 - x_2 + 2x_3 = -4$$

Question asked required the candidates to express the system of equations into matrix form as under,

$$\begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & -2 \\ -3 & -1 & 2 \end{bmatrix}\begin{bmatrix} x_2 \\ x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix}$$

$\Rightarrow AX = B \quad \Rightarrow X = A^{-1}B$

They were required to state that $A^{-1} = \frac{1}{|A|}$ adj A

Candidates were expected to calculate $A^{-1}$ by calculating the determinate $|A| = 10$ and adj A and substituting their values as under,

$$\text{adj A} = \begin{bmatrix} 0 & -5 & -5 \\ 4 & 13 & 7 \\ 2 & -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj A} = \frac{1}{10}\begin{bmatrix} 0 & -5 & -5 \\ 4 & 13 & 7 \\ 2 & -1 & 1 \end{bmatrix}$$

Candidates were required to expressed $X = A^{-1}B$ and calculate the values of X as under,

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 0 & -5 & -5 \\ 4 & 13 & 7 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 0 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{8}{5} \\ \frac{1}{5} \end{bmatrix}$$

They were required to give the solution $x_1 = 2, \quad x_2 = -\frac{8}{5}, \quad x_3 = \frac{1}{5}$

*Question asked was of mixed cognitive levels (knowledge, understanding and application) which required the candidates to recall the procedure for expressing the system of equations in the form of matrices and then by finding inverse of the matrix, comprising the coefficients of ($x_1$, $x_2$ and $x_3$), to calculate their values. This question was attempted by around 33.79 percent of the candidate and out of them some (25-30%) candidates faced difficulty in calculating the inverse of the matrix. They on average secured 4.02 (50.31%) marks and around 15.17 percent out of them secured full (100%) marks in this question.*

**Question 4**

Solve the system of equations

$x^2 - y^2 = 5$

$4x^2 - 3xy = 18$

This question required the candidates to multiply the first equation by 18 and second by 5 and then subtracting first equation from the second as under,

$18x^2 - 18y^2 = 90$

$20x^2 - 15xy = 90$

$\Rightarrow 2x^2 - 15xy - 18y^2 = 0$

Candidates were expected to factorize the equation to calculate x in terms of y as under,

$(x-6)\ (2x-3y)=0$

$\Rightarrow$ either $x-6y=0$ or $\Rightarrow 2x-3y=0$

$\Rightarrow$ either $x=6y$ or $x=\frac{3y}{2}$

They were required to calculate the solution sets by substituting the value of x in equation as under,

$x^2-y^2=5$

SYSTEM – I, $x=6y \Rightarrow \left(\pm\frac{6}{\sqrt{7}}, \pm\frac{1}{\sqrt{7}}\right)$

SYSTEM – II, $x=\frac{3y}{2} \Rightarrow (\pm 6, \pm 2)$

*Question asked was of **knowledge-cum-understanding level** which required the candidates to combine the given two equations by eliminating the constant terms. They were expected to factorize the resultant equation and by equating each factor equal to zero separately to calculate the required values of 'x' and 'y'. Almost 47.18 percent of the candidates who attempted this question answered it correctly while 30-35 percent out of them gave vague, incorrect and incomplete answers. Some (10-15%) out of these candidates tried to solve this question by applying an alternate method but majority of them faced difficulty in correctly finding the solution set as required. They on the average secured 3.64 (45.51%) marks and around 6.21 percent out of them secured full (100%) marks in this question.*

**Question 5**

If the numbers $\frac{1}{2}, \frac{4}{21}$ and $\frac{1}{36}$ are subtracted from three consecutive terms of G.P. the resulting numbers are in H.P. Find the numbers if their product is $\frac{1}{27}$

This question required the candidate to suppose that $\frac{a}{r}$, a, ar, are the numbers of GP

They were expected to use the given data for product of numbers as under,

$\left(\frac{a}{r}\right)(a)(ar)=\frac{1}{27} \Rightarrow a^3=\frac{1}{27} \Rightarrow a=\frac{1}{3}$

Candidates were expected to subtract the given numbers to obtain H.P. as under,

$H.P: \frac{a}{r}-\frac{1}{2}, a-\frac{4}{21}, ar-\frac{1}{36}$

$\Rightarrow \frac{2a-r}{2r}, \frac{21a-4}{21}, \frac{36ar-1}{36}$

They were required to take the reciprocal of the terms to get A.P. as under,

$\Rightarrow$ A.P: $\frac{2r}{2a-r}, \frac{21}{21a-4}, \frac{36}{36ar-1}$

Candidates were expected to use the property of A.P. where common difference occurs between consecutive terms as under,

$\Rightarrow \frac{36}{36ar-1}-\frac{21}{21a-4}=\frac{21}{21a-4}-\frac{2r}{2a-r}$

They were required to substitute value of $a=\frac{1}{3}$ to calculate the value of 'r' as under,

$$\frac{36}{12r-1} - 7 = 7 - \frac{6r}{2-3r} \Rightarrow 144\,r^2 - 123r - 25 = 0$$

$$\Rightarrow r = \frac{25}{48}, \frac{1}{3}$$

The candidates were expected to construct the G.P. as under;

When $a = \frac{1}{3}$, $r = \frac{25}{48}$ or when $a = \frac{1}{3}$, $r = \frac{1}{3}$

Ist Number $= \frac{a}{r} = \frac{16}{25}$ Ist Number $= \frac{a}{r} = 1$

2nd Number $= a = \frac{1}{3}$ 2nd Number $= a = \frac{1}{3}$

3rd Number $= ar = \frac{25}{144}$ 3rd Number $= ar = \frac{1}{9}$

Candidates were asked to give the first three numbers of possible G.P. series as;

$\frac{16}{25}, \frac{1}{3}, \frac{25}{144}$ or $1, \frac{1}{3}, \frac{1}{9}$

*Question asked was of mixed cognitive levels (knowledge, understanding and application) which required the candidates to recall the properties of different types of series and by using them and the information contained in the question to find the required G.P. series. This question was attempted by around 21.55 percent of the candidates. However, around 25-30 percent out of them faced difficulty in finding the value of "r". Candidates on average secured 5.56 (69.48%) marks and around 46.60 percent out of them secured full (100%) marks in this question.*

**Question 6**

Identify the following series and find its sum

$$1 - \frac{1}{2}\left(\frac{1}{2}\right) - \frac{1.3}{2.4}\left(\frac{1}{2}\right)^2 - \frac{1.3.5}{2.4.6}\left(\frac{1}{2}\right)^3 + \ldots\ldots$$

This question required the candidates to suppose that the given series (A) is identical to one of the general series (B) as under,

$$S = 1 - \frac{1}{2}\left(\frac{1}{2}\right) - \frac{1.3}{2.4}\left(\frac{1}{2}\right)^2 - \frac{1.3.5}{2.4.6}\left(\frac{1}{2}\right)^3 + \ldots\ldots \text{(A)}$$

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \cdots \text{(B)}$$

They were expected to determine the values of 'n' and 'x' by comparing the terms of x on the R.H.S. of A & B and as under,

$$n\,x = -\frac{1}{4}; \quad n\,(n-1)\,x^2 = \frac{3}{16}$$

$$n = -\frac{1}{2}, \quad x = \frac{1}{2}$$

They were required to compare the L.H.S. of equations A & B and by substituting the values of n and x to calculate the required sum of the series as under,

$\Rightarrow S = (1+x)^n \Rightarrow S = \left(1+\frac{1}{2}\right)^{-\frac{1}{2}}$

$\Rightarrow S = \sqrt{\frac{2}{3}}$

*Question asked was of **knowledge-cum-understanding level** which required the candidates to recall the standard series in the form of $(1+x)^n$ and by comparing the coefficients of x in the two series (given and standard) to calculate the values of 'x' and 'n'. Candidates were also required to give the series by substituting the values of 'x' and 'n' in supposed standard series. This was one of the low choice questions from this section which was attempted by almost 38.28 percent of the candidates who on average secured 6.28 (78.48%) marks and around 58.47 percent out of them secured full (100%) marks in this question.*

**Question 7**

Find the values of $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ if $\tan\alpha = \frac{-15}{8}$ and $\sin\beta = \frac{-7}{25}$, neither $\alpha$ nor $\beta$ lie in the 4th quadrant.

This question required the candidates to argue that since $\alpha$ and $\beta$ do not lie in 4th quadrant but $\tan\alpha = \frac{-15}{8}$ and $\sin\beta = \frac{-7}{25}$ so these angles must lie in the 2nd and 3rd quadrants respectively.

They were required to state and use the trigonometric identities to find the value of cos $\alpha$, $\sin\alpha$ and $\cos\beta$ as under,

$\sec^2\alpha = 1 + \tan^2\alpha \Rightarrow \sec^2\alpha = \frac{289}{64} \Rightarrow \sec\alpha = \pm\frac{17}{8}$

$\Rightarrow \cos\alpha = -\frac{8}{17} \quad \because \quad \alpha \in$ Quad – II

$\sin^2\alpha = 1 - \cos^2\alpha \Rightarrow \sin^2\alpha = \frac{225}{289} \Rightarrow \sin\alpha = \pm\frac{15}{17}$

$\therefore \sin\alpha = \frac{15}{17} \quad \because \quad \alpha \in$ Quad – II

$\cos^2\beta = 1 - \sin^2\beta \Rightarrow \cos^2\beta = \frac{576}{625} \Rightarrow \cos\beta = \pm\frac{24}{25}$

$\therefore \cos\beta = -\frac{24}{25} \quad \because \quad \beta \in$ Quad – III

Candidates were required to state the trigonometric identities for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ and to calculate their values by substituting the values of cos $\alpha$, $\sin\alpha$, $\sin\beta$ and $\cos\beta$ as under,

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha+\beta) = \left(\frac{15}{17}\right)\left(-\frac{24}{25}\right) + \left(-\frac{8}{17}\right)\left(-\frac{7}{25}\right) = \frac{304}{425}$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = \left(-\frac{8}{17}\right)\left(-\frac{24}{25}\right) + \left(\frac{15}{17}\right)\left(-\frac{7}{25}\right) = \frac{297}{425}$

Candidates were expected to summarize the results as under;

$\sin(\alpha+\beta) = \frac{304}{425}$, $\cos(\alpha+\beta) = \frac{297}{425}$

*Question asked was of mixed cognitive levels (knowledge, understanding and application) which required the candidates to infer from the given data that the angles $\alpha$ and $\beta$ lie in the second and third quadrants respectively and by using the given data and trigonometric identities calculated the values of cos $\alpha$, sin $\alpha$ and cos $\beta$. By using the calculated values found the values of $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ as required. This was one of the low choice questions attempted by around 41.53 percent of the candidates. They secured on the average 4.86 (60.71%) marks and around 35.26 percent out of them secured full (100%) marks in this question.*

**Question 8**

Prove that $\cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5}$

This question required the candidates to apply trigonometric identity on the L.H.S. of equation and simplify it as under,

$$\cos^{-1}\frac{63}{65} + \tan^{-1}\frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2} = \sin^{-1}\frac{3}{5}$$

$$\cos^{-1}\frac{63}{65} + \tan^{-1}\frac{5}{12} = \sin^{-1}\frac{3}{5}$$

They were required to suppose $\alpha = \cos^{-1}\frac{63}{65}$ iff $\cos\alpha = \frac{63}{65}$ where $\alpha \in [0,\pi]$

The candidates were required to to find the value of sin $\alpha$ by using trigonometric identity and substituting the value of cos $\alpha$ as under,

$$\sin\alpha = +\sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{3969}{4225}} = \frac{16}{65}$$

They were also required to suppose $\beta = \tan^{-1}\frac{5}{12}$ iff $\tan\beta = \frac{5}{12}$ where $\beta \in \left[\frac{\pi}{2}, \pi\right]$

They were expected to find the value of cos $\beta$ and sin $\beta$ by using trigonometric identities and by substituting the value of $\tan\beta$ as under,

$$\sec^2\beta = 1 + \tan^2\beta = 1 + \frac{25}{144} = \frac{169}{144}$$

$$\Rightarrow \cos^2\beta = \frac{144}{169} \Rightarrow \cos\beta = \frac{12}{13}$$

$$\sin^2\beta = 1 - \cos^2\beta = 1 - \frac{144}{169} = \frac{25}{169} \Rightarrow \sin\beta = \frac{5}{13}$$

Candidates were required to use trigonometric identity to find value of sin $(\alpha + \beta)$ as under,

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha+\beta) = \left(\frac{16}{65}\right)\left(\frac{12}{13}\right) + \left(\frac{63}{65}\right)\left(\frac{5}{13}\right) = \frac{3}{5}$$

They were required to prove by simplifying L.H.S. of equation equal to the R.H.S. of the given equation as under,

$$\alpha+\beta = \sin^{-1}\frac{3}{5} \Rightarrow \cos^{-1}\frac{63}{65} + \tan^{-1}\frac{5}{12} = \sin^{-1}\frac{3}{5}$$

$$\Rightarrow \cos^{-1}\frac{63}{65} + 2\tan^{-1}\frac{1}{5} = \sin^{-1}\frac{3}{5} \quad \because 2\tan^{-1}\frac{1}{5} = \tan^{-1}\frac{5}{12}$$

*Question asked was of mixed cognitive levels* ***(knowledge, understanding and application)*** *which required the candidates to apply trigonometric identities to find the value of sin* $(\alpha+\beta)$ *and by using it to prove the required relation. This was the most popular choice question attempted by around 65.69 percent of the candidates who on average secured 4.85 (60.67%) marks and around 33.28 percent out of them secured full (100%) marks in this question.*

**Question 9**

Show that the set consisting of elements of form

$\{a+\sqrt{3}\,b \ \ (a,b \text{ being rational})\}$ is an abelian group w.r.t addition

This question required the candidates to state the following properties by supposing,

G = $\{a+\sqrt{3}\,b \mid a,b \in Q\}$

Closure Property

If, $a_1+\sqrt{3}\,b_1$, $a_2+\sqrt{3}\,b_2 \in G$

Then, $(a_1+a_2)$, $\sqrt{3}\,(b_1+b_2)$, $\in G$

Associative Property

If, $a_1+\sqrt{3}\,b_1$, $a_2+\sqrt{3}\,b_2$, $a_3+\sqrt{3}\,b_3 \in G$

then $(a_1+\sqrt{3}\,b_1)+(a_2+\sqrt{3}\,b_2)+(a_1+\sqrt{3}\,b_3)$

$= (a_1+a_2+a_3)+\sqrt{3}(b_1+b_2+b_3)$

Identity Element

Candidates were required to state that for any element $a+\sqrt{3}\,b$ there exists an element $0+\sqrt{3}\,0$, such that

$a+\sqrt{3}\,b + 0+\sqrt{3}\,0 = a+\sqrt{3}\,b \in G$

where $0+\sqrt{3}\,0$ is called an Additive Identity in G

Inverse Element

They were also required to state that for any element $a+\sqrt{3}\,b$ there exists an element $-a-\sqrt{3}\,b$, such that

$a+\sqrt{3}\,b+(-a-\sqrt{3}\,b) = 0+\sqrt{3}\,0$

Both $(a+\sqrt{3}\,b)$ and $(-a-\sqrt{3}\,b)$ are additive inverse of each other

Commutative Property

Candidates were expected to state that if $a_1+\sqrt{3}\,b_1$, $a_2+\sqrt{3}\,b_2 \in G$ then

$(a_1+\sqrt{3}\,b_1)+(a_2+\sqrt{3}\,b_2)=(a_2+\sqrt{3}\,b_2)+(a_1+\sqrt{3}\,b_1)$

$\Rightarrow (a_1+a_2)+\sqrt{3}(b_1+b_2)=(a_1+a_2)+\sqrt{3}(b_1+b_2)$

They were required to state that the G is an abelian group under addition

*Question asked was of* ***knowledge-cum-understanding level*** *in which candidates were required to recall the properties of abelian group under addition and apply them to prove that the elements of the given form are abelian group under addition. This was one of the low*

*choice questions attempted by around 50 percent of the candidates who on the average secured 4.57 (57.07%) marks and around 20.68 percent out of them secured full (100%) marks in this question.*

**Summary**

The higher facility and positive discrimination indices suggest that the paper was reasonably good and candidates performed quite well in answering the questions. The positive higher value of discrimination index reflects that those students who were overall in the upper bracket performed better in all questions as compared to those who were in the lower bracket. Since all the questions were given from the prescribed book, which are normally covered in classroom teaching as a routine, so the students had to mere recall their solutions (knowledge level). Those students who had spent more time in preparation performed better in all the questions. The question-wise examinees falling in different quintiles are given in the figure 3 and percentage of those securing 100 percent marks in each question are given in figure 4 below:

Fig. 3a Section-wise Quartile Distribution of Examinees

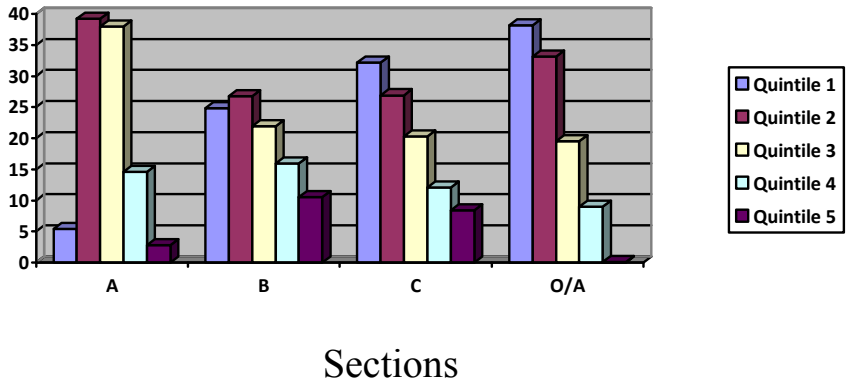


Fig. 3b Question-wise Quartile Distribution of Examinees

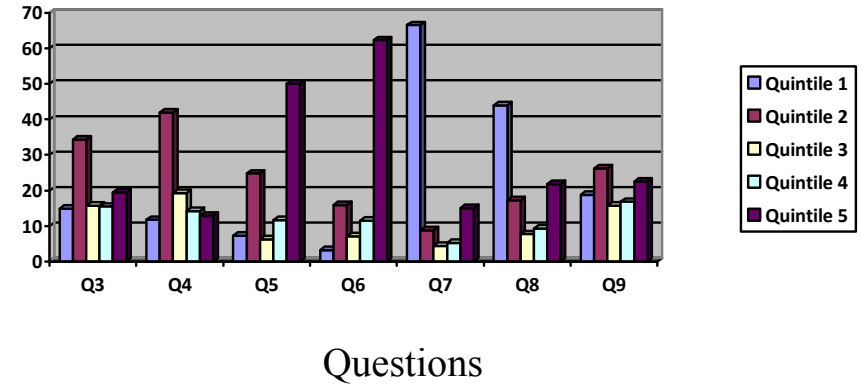



Figure 4 Question-wise percentage of candidates securing full (100%) marks.